Principled Bayesian Inference for Estimating Globular Cluster Counts in Ultra-Diffuse Galaxies using Mark-Dependently Thinned Point Process

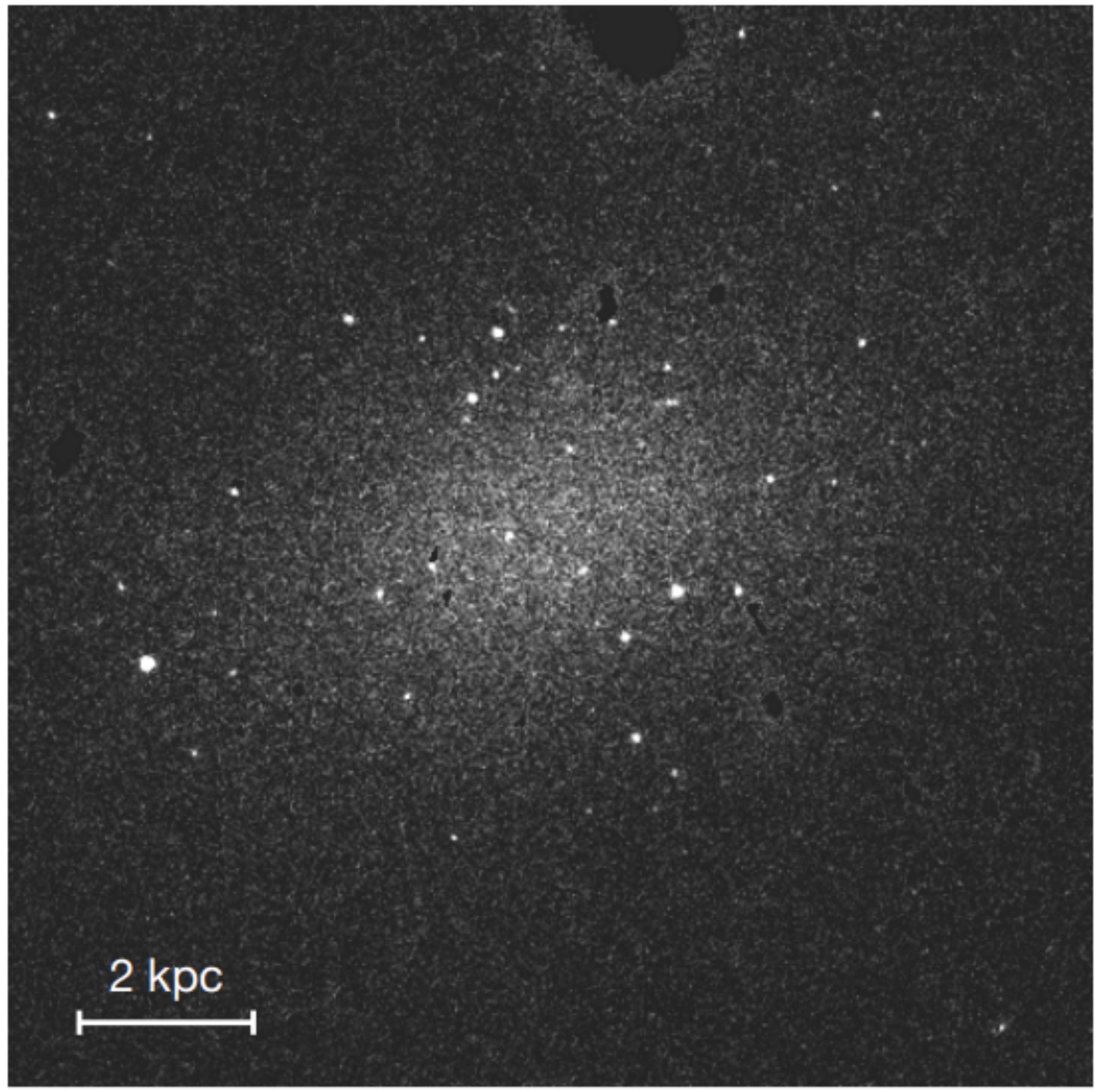
CHASC Meeting, July 21, 2023

David Dayi Li

Join work with: Patrick Brown, Gwendolyn Eadie, William Harris, Roberto Abraham, Joshua Speagle, Samantha Berek, Shany Danieli

Ultra-Diffuse Galaxies (UDGs)

- A class of low-surface brightness galaxies first detected in abundance with Dragonfly (van Dokkum et al. 2015) in 2015
- Dragonfly-44:



Ultra-Diffuse Galaxies

- Many UDGs have GCs despite their low-surface brightness
- Examples: Dragonfly-44, NGC1052-DF2, etc.
- GCs in UDGs —> Inferring Dark Matter content
- But..
- Massive dispute on the actual count estimates, e.g., NGC 1052-DF2
- Existing methods on GC counts have issues

How to count?

- Compact sources
- Aperture correction
- GCLF correction
- Background correction

Issues of GC Counting in UDGs

- What is a GC?
- GC membership?
- Globular cluster luminosity function (GCLF): universal or no?
- Uncertainty of GCs near/below detection limit
- Correction of GCs at large radii
- Crowding
- Abnormal GCLF (NGC 1052-DF2 and DF4)

Issues of Current Methods

- Weirdest of them all:
- Some UDGs get negative estimates....

Issues of Current Methods

- Fragmented Analysis
- Ad-Hoc
- Uncertainty of assumptions made
- Newer methods: Amorisco et al. 2018, Carlsten et al, 2022
- Lacks a principled method

Point Process

- Point process $\mathbf{X}$: stochastic process consists of random points.
- A realization $\mathbf{x} = \{x_1, \ldots, x_n\} \subset S \subset \mathbb{R}^2$.
- Intensity function: $\lambda(s) \geq 0, \ s \in S$
- λ: mean surface number density
- Homogenous Poisson process: constant λ
- If λ depends on location s, then an inhomogeneous Poisson process (IPP).
- Want to model GC point pattern as an IPP
- **Big issue though**: some GCs are not observed...

Thinned Poisson Process (TPP)

- TPP: randomly removing some points from a point process.
- Goodies:
- If the original process is Poisson, and the removal is independent.
- The thinned process is still Poisson.
- If the original process has intensity $\lambda(s)$, and thinning probability is $p(s)$. The thinned process has intensity $\lambda(s)p(s)$.
- But GCs removed based on their magnitudes M

Marked Point Process

- Marked point process $(\mathbf{X}, \mathscr{M})$: each point has some characteristic (mark) attached
- Realization $(\mathbf{x}, \mathbf{M}) = \{(x_1, M_1), \ldots, (x_n, M_n)\} \subset D \subset \mathbb{R}^2 \times \mathbb{R}$
- Intensity $\lambda(s, m) \geq 0, \ (s, m) \in D$.
- In our case, mark is the magnitude of GCs.
- Decomposition of intensity:

$$\lambda(s, m) = \lambda_0(s)\pi_0(m \mid s).$$

Mark-Dependently Thinned Point Process (MTPP)

- Myllymaki, 2009 proposed the MTPP: thinning depends on the mark
- But only theoretical: no consideration of joint modelling of location and marks
- Mark-location thinning probability: $t(s, m)$
- Want the thinned intensity $\lambda_t(s, m) = \lambda_{t,0}(s)\pi_t(m \mid s)$.
- π_t: truncation applied to π_0 based on $t(s, m)$:

$$\pi_t(m \mid s) = \frac{\pi_0(m \mid s)t(s, m)}{\int \pi_0(m \mid s)t(s, m)dm}.$$

MTPP

- $\lambda_{t,0}(s)$: seek a thinning probability $\rho(s)$ that only depends on s
- Marginalize the effect of m:

$$\rho(s) = \int \pi_0(m \mid s) t(s, m) dm,$$

$$\lambda_{t,0}(s) = \lambda_0(s) \rho(s),$$

and

$$\lambda_t(s, m) = \lambda(s, m) t(s, m).$$

Models

- Assuming different GCLF:

$$\mathbf{X} \mid \Lambda \sim \mathrm{IPP}(\Lambda),$$

$$\Lambda(s) = \sum_{m=1}^{N} \lambda_m(s), \text{ (superposition of } N \text{ environments)}$$

$$M_i \mid x_i, \Lambda \sim \sum_{m=1}^{N} p_m(x_i) \boldsymbol{\nu}_f(\mu_m, \sigma_m^2), \ i = 1, \ldots, n$$

$$p_m(s) = \lambda_m(s)/\Lambda(s),$$

$$\lambda_1(s) = \beta p(s) \Psi_f(\mu_1, \sigma_1^2), \text{ (IGM GCs)}$$

$$\lambda_m(s) = \mathrm{Sersic}(s; N_m, R_m, n_m) p(s) \Psi_f(\mu_m, \sigma_m^2), \ m \geq 2. \text{ (GCs in galaxies)}$$

- Assuming the same GCLF:

$$\mathbf{X} \mid \Lambda \sim \text{IPP}(\Lambda),$$

$$\Lambda(s) = \sum_{m=1}^{N} \lambda_m(s),$$

$$M_i \mid x_i, \Lambda \sim \boldsymbol{\nu}_f(\mu, \sigma^2), \ i = 1, \ldots, n$$

$$\lambda_1(s) = \beta p(s) \Psi_f(\mu, \sigma^2),$$

$$\lambda_m(s) = \text{Sersic}(s; N_m, R_m, n_m) p(s) \Psi_f(\mu, \sigma^2), \ m \geq 2.$$

- $\boldsymbol{\nu}_f(\mu, \sigma^2)$: marginal distribution of truncated noisy magnitude with density ($\pi_t(m \mid s)$):

$$\psi_f(x; \mu, \sigma^2) = \frac{\psi(x, \mu, \sigma^2) f(x)}{\int_{-\infty}^{\infty} \psi(x, \mu, \sigma^2) f(x) dx} \equiv \frac{\psi(x, \mu, \sigma^2) f(x)}{\Psi_f(\mu, \sigma^2)},$$

- $t(s, m) = p(s) f(m)$
- $f(m) = 1/(1 + \exp(a(m - m_0)))$
- $\psi(x, \mu, \sigma^2)$: marginal density of non-truncated noisy magnitude ($\pi_0(m \mid s)$):

$$\psi(x, \mu, \sigma^2) = \int_{-\infty}^{\infty} \phi(x; t, \sigma^2(t)) \phi(t; \mu, \sigma^2) dt.$$

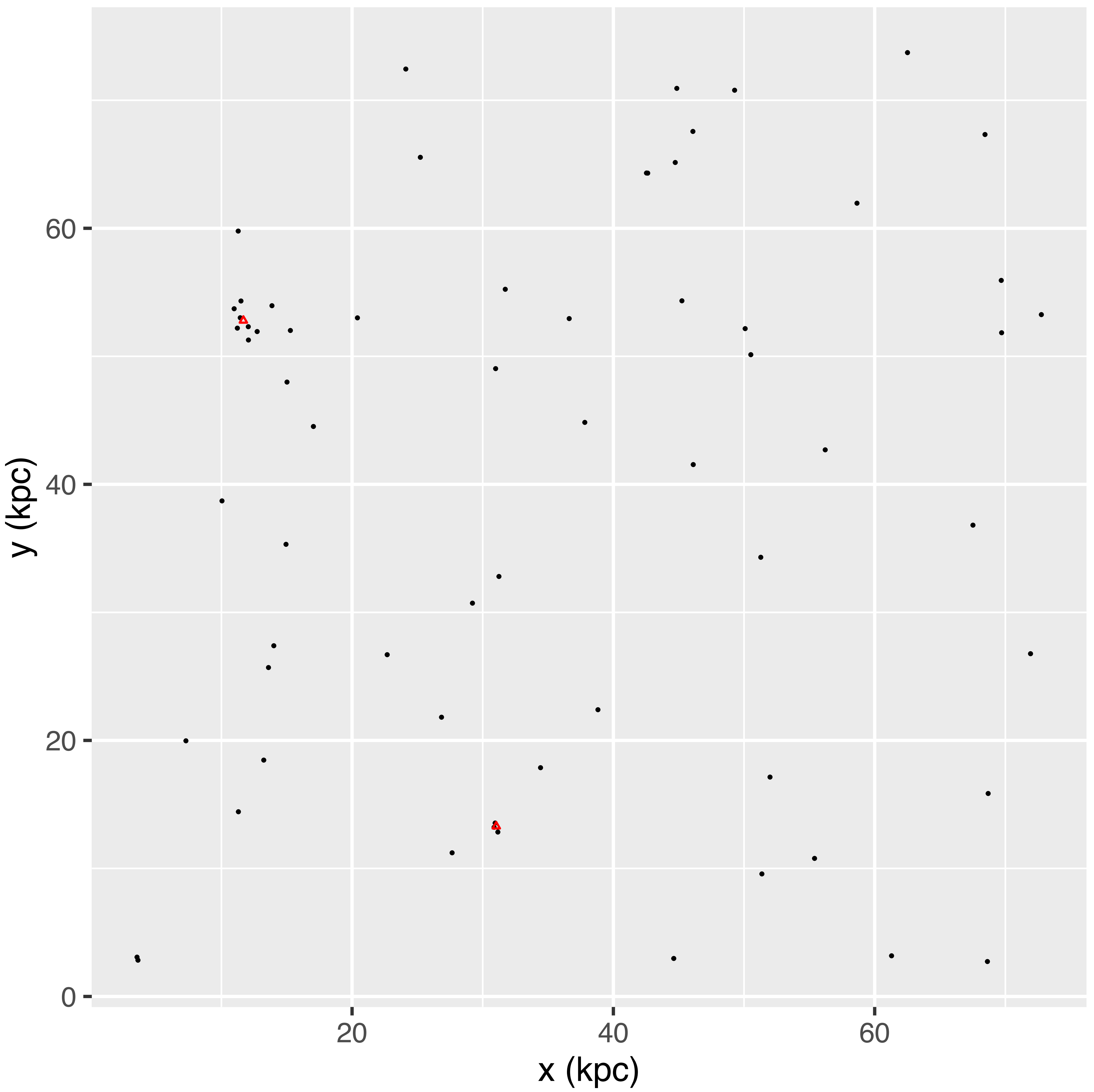
- $\sigma(m_{\text{true}}) = \beta_0 \exp(\beta_1(m_{\text{true}} - m_1))$

Results (Preliminary)

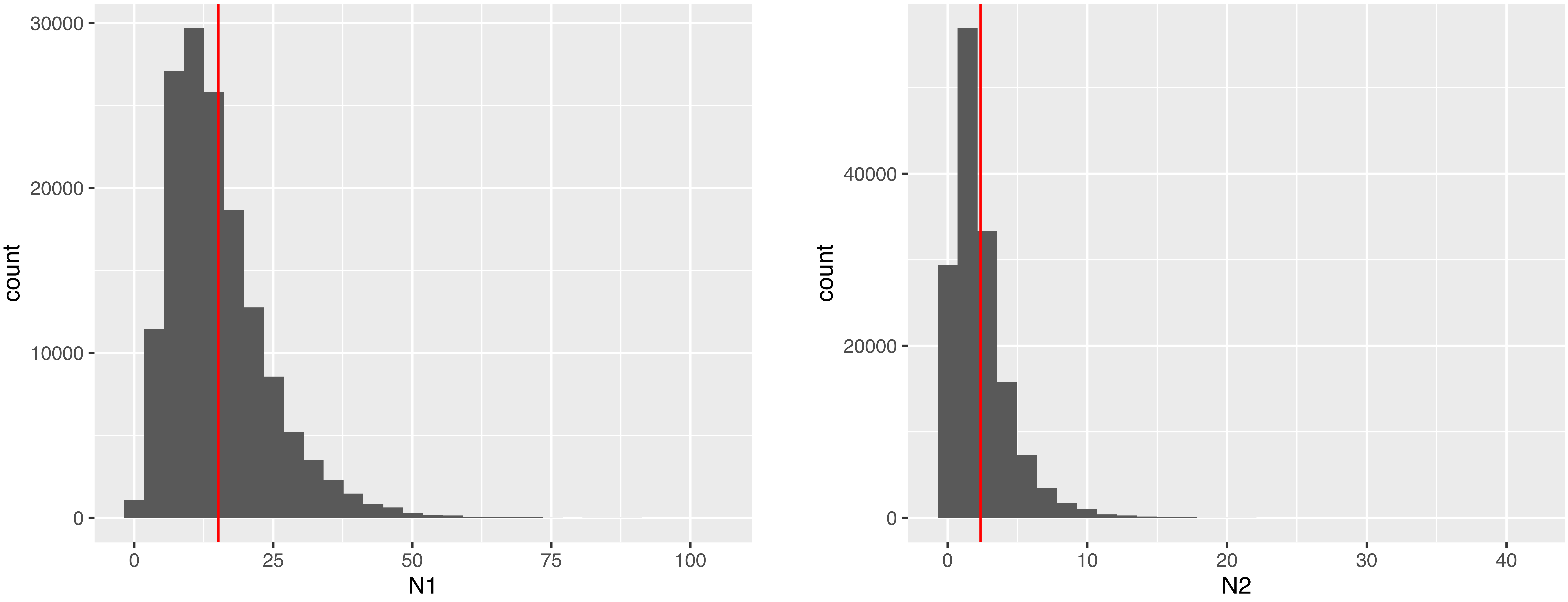
- Data: PIPER survey (Harris et al., 2020): HST targeting the GC population in the Perseus galaxy cluster
- Consider the magnitude in F814W filter

Results

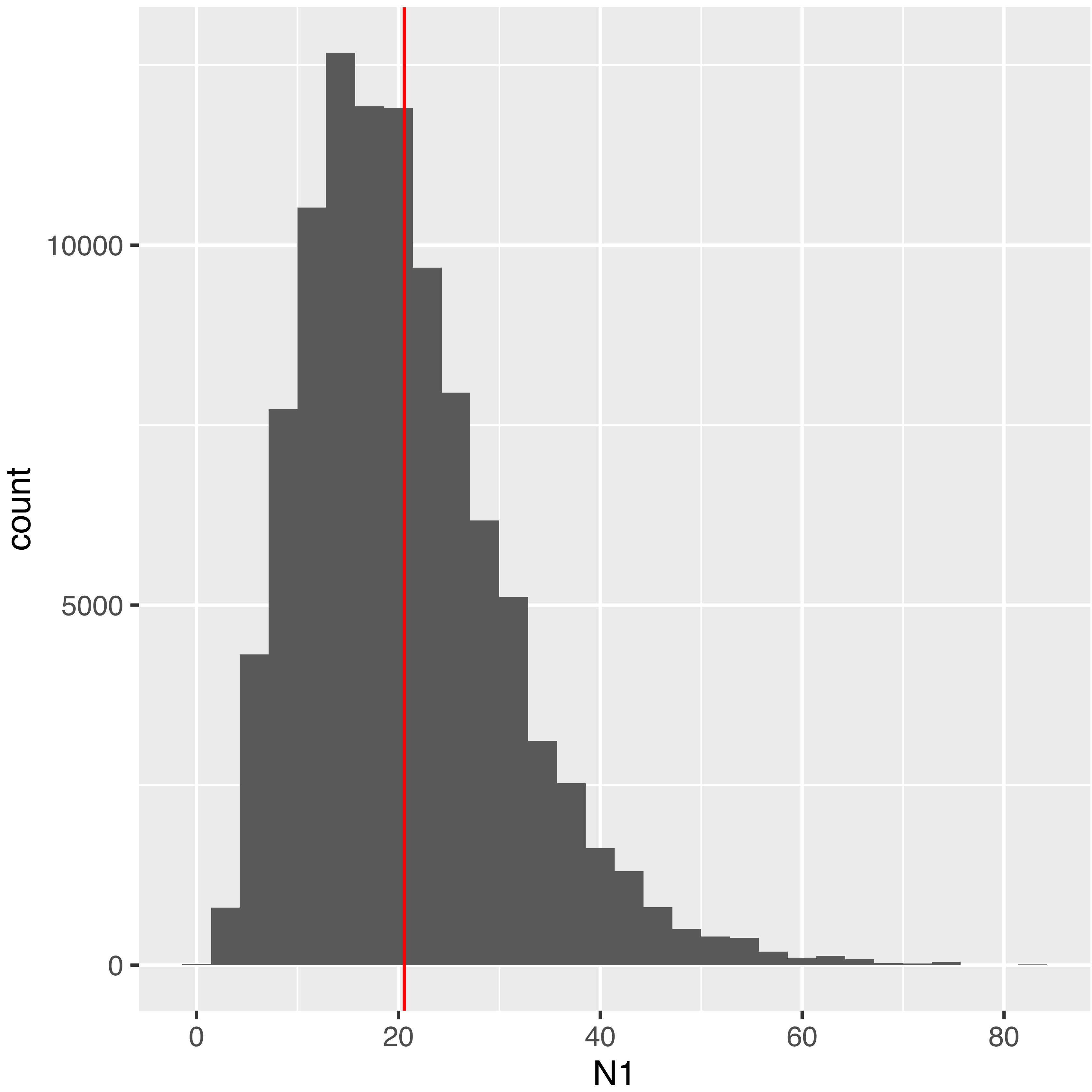
- v7-ACS field:

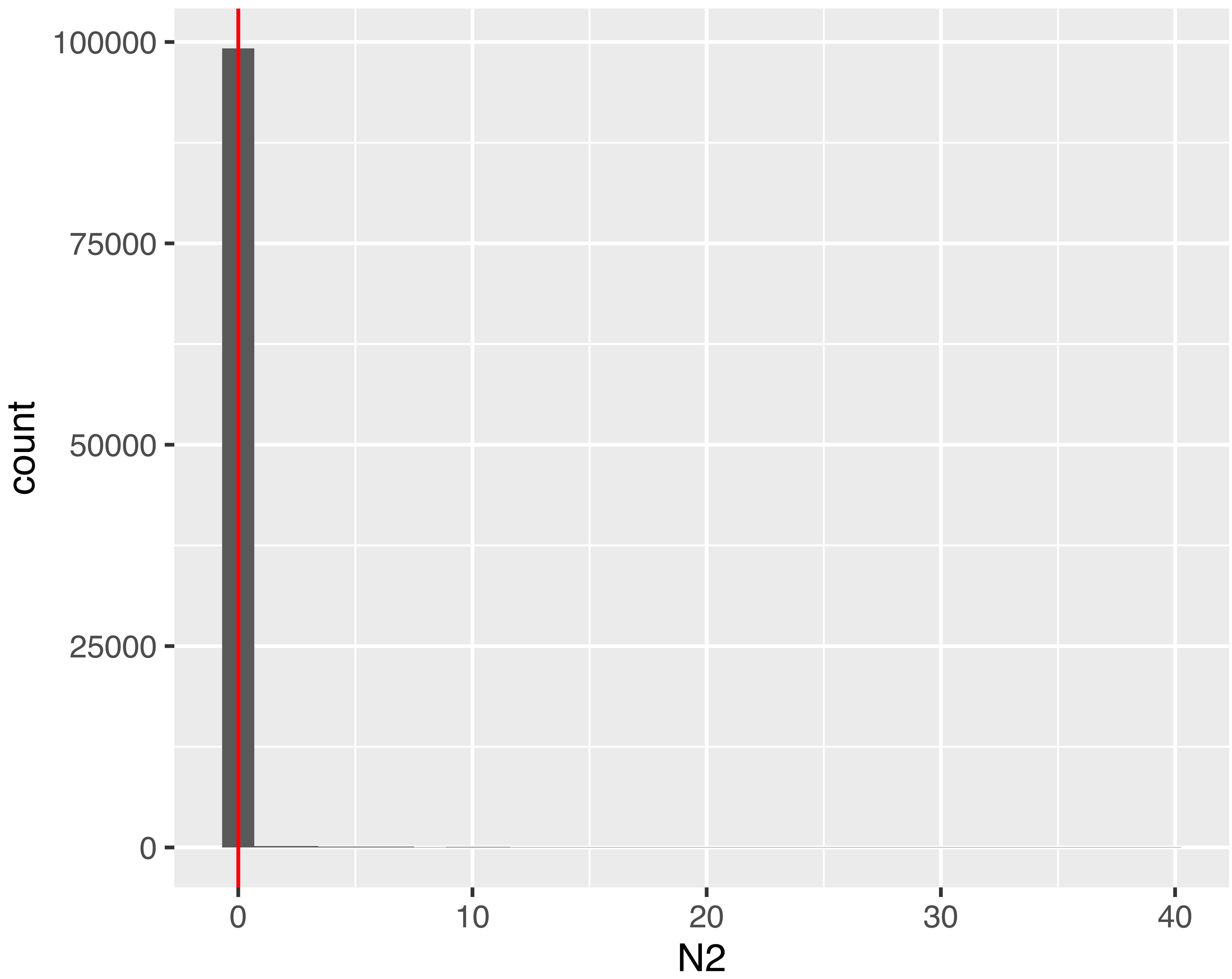


- Posterior of the number of GCs in the two UDGs (Model 1):

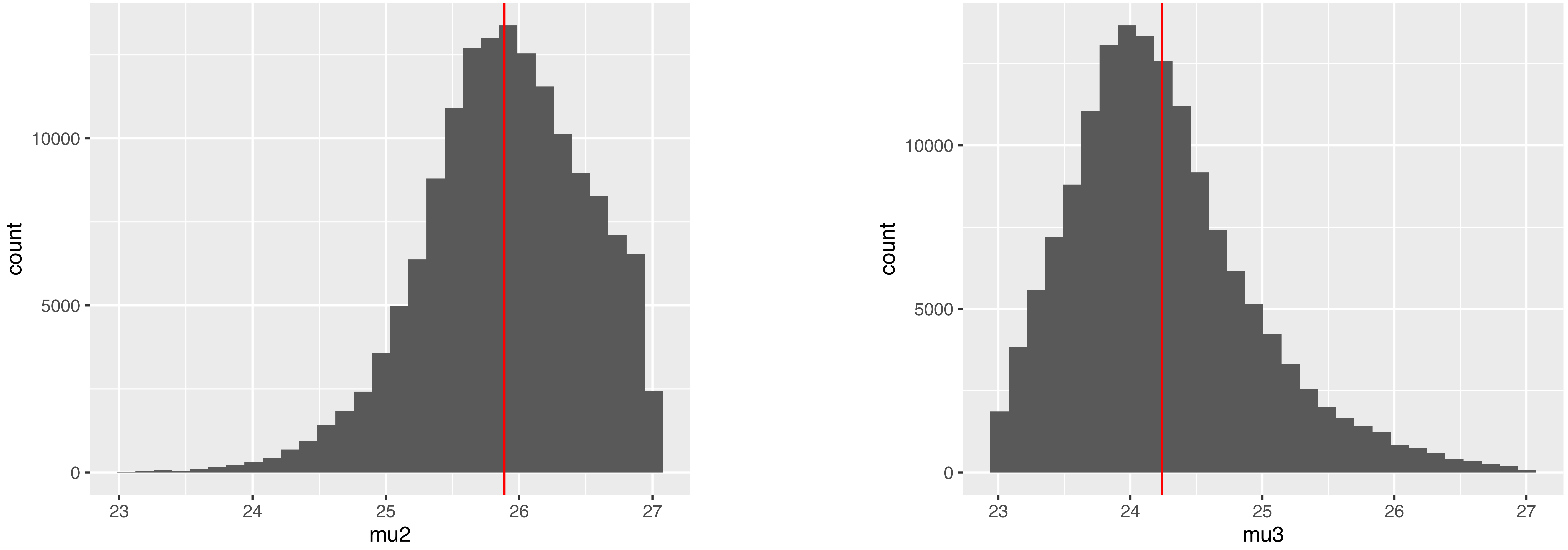


- Posterior of the number of GCs in the two UDGs (Model 2):

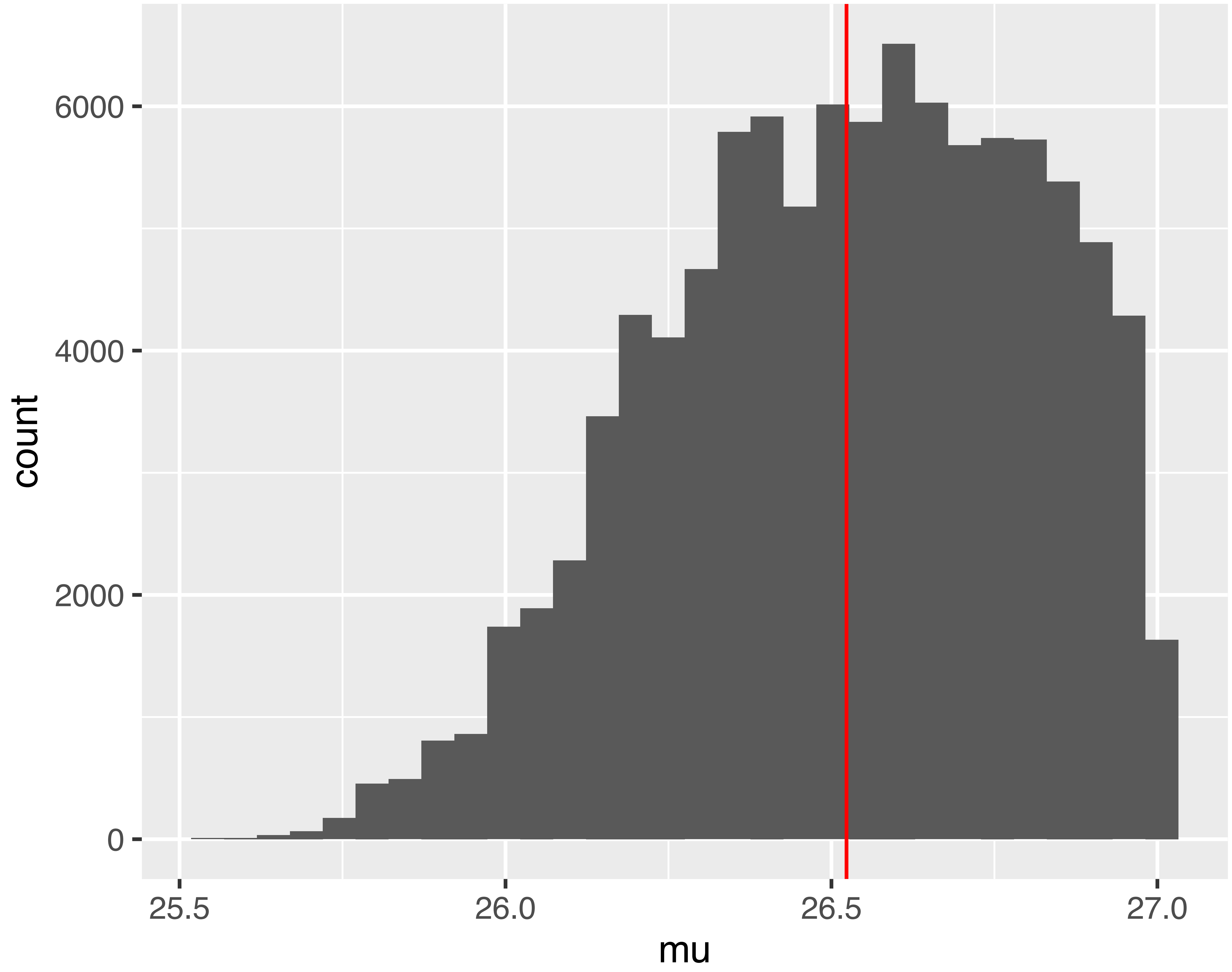




- Posterior of the mean of GCLF (Model 1):



- Posterior of the mean of GCLF (Model 2):



Some Comparison

- Our estimates vs. Previously obtained estimates with traditional method (by Steven Janssens)

	Mean	Median	.025 quantile	.975 quantile
N1 (model 1)	15.12	13.82	3.138	37.83
N2 (model 1)	2.35	1.805	0.128	7.88
N1 (model 2)	20.59	19.04	5.76	44.96
N2 (model 2)	0.05	2.18×10^{-6}	1.03×10^{-6}	1.05×10^{-6}
N1 (traditional)	$14.19 \pm^{1.28}_{1.47}$	NA	NA	NA
N2 (traditional)	$2.26 \pm^{1.12}_{0.62}$	NA	NA	NA

Thank You!